

# Sinusoidally Modulated Vacuum Rabi Oscillation of a Two-Level Atom in an Optomechanical Cavity

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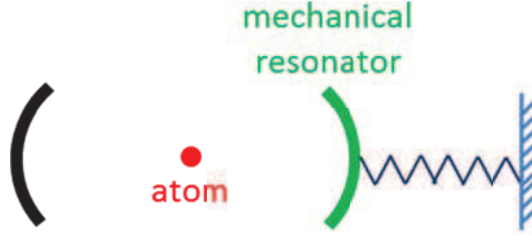
**Abstract.** We study the coherent dynamics of an excited two-level atom in a vacuum optomechanical cavity and find that the original atom-cavity Rabi oscillation is sinusoidally modulated by the light-mechanics coupling as the Rabi splitting is on resonance with the mechanical mode. We develop an analytic model in a three-dimensional Hilbert subspace to explain this phenomenon and employ numerical simulations of the density-matrix master equation to confirm our analysis. We also show that the modulated Rabi oscillation survives in presence of dissipations and other non-ideal factors.

## 1. Introduction

Optomechanics and cavity quantum electrodynamics (QED) both study light-matter interactions. While cavity QED involves light interacting with nearly-resonant systems such as atoms [1, 2], optomechanics treats highly off-resonant couplings between light and mechanical objects [3, 4, 5, 6, 7]. The rapid developments in optomechanics towards the strong-coupling regime make it possible to explore the quantum effects of single-photons on macroscopic resonators [8, 9, 10, 11, 12, 14, 13], and hybrid systems consisting of optomechanics and cavity QED components are expected to exhibit rich and novel features in the quantum regime [15, 16, 17, 18].

As is well known, an excited two-level atom in a vacuum cavity undergoes Rabi oscillation at a frequency proportional to the atom-photon coupling. This is one excellent example illustrating how the cavity modifies the optical properties of the atom, since in free space the atom would simply decay exponentially. Adding a mechanical resonator to the atom-cavity system via optomechanical coupling further changes the atomic behavior, even though the mechanical resonator does not directly interact with the atom. In this paper, we study the coherent dynamics of an excited two-level atom in a vacuum optomechanical cavity. We find that as the atom-cavity Rabi splitting is on resonance with the mechanical mode, the original Rabi oscillation is *sinusoidally modulated* by the

optomechanical (light-mechanics) interaction. This sinusoidal modulation of the Rabi oscillation is non-trivial since it occurs in absence of any initial photons and phonons. An analytic model in a *three-dimensional* Hilbert subspace of the hybrid system is developed to explain this intriguing phenomenon, and numerical simulations of the density-matrix master equation are employed to confirm the analytical analysis and to demonstrate that the result qualitatively holds even as dissipations and other non-ideal factors are taken into account.



**Figure 1.** Schematic of a two-level atom inside an optomechanical cavity.

## 2. Model and Hamiltonian

We consider the setup in Fig. 1, where a two-level atom is placed in a cavity with a movable end mirror as the mechanical resonator. The single optical cavity mode is coupled to both the atom (via dipole-moment interaction) and the mechanical resonator (via radiation pressure). The hybrid atom-optomechanical system is described by the Hamiltonian ( $\hbar = 1$ ) [16]

$$\hat{H} = \omega_c \hat{c}^\dagger \hat{c} + \frac{\omega_a}{2} \hat{\sigma}_z + g_{ca} (\hat{\sigma}_+ \hat{c} + \hat{\sigma}_- \hat{c}^\dagger) + \omega_m \hat{b}^\dagger \hat{b} - g_{cm} \hat{c}^\dagger \hat{c} (\hat{b} + \hat{b}^\dagger). \quad (1)$$

Here  $\hat{c}$  and  $\hat{b}$  are the annihilation operators for the optical field (of frequency  $\omega_c$ ) and the mechanical resonator (of frequency  $\omega_m$ ), respectively,  $\hat{\sigma}_z = |e\rangle_a \langle e| - |g\rangle_a \langle g|$ ,  $\hat{\sigma}_+ = \hat{\sigma}_-^\dagger = |e\rangle_a \langle g|$ , with  $|e\rangle_a$  ( $|g\rangle_a$ ) as the excited (ground) state of the atom,  $\omega_a$  is the transition frequency between the atomic states, and  $g_{cm}$  ( $g_{ca}$ ) characterizes the strength of the light-mechanics (light-atom) interaction. In this letter, we use subscripts “c”, “a”, “m” to respectively denote “optical (cavity)”, “atomic” and “mechanical” states or parameters. The dimension of the hybrid-system Hilbert space is infinite. However, as we will show later, if the system is initially in  $|0\rangle_c |e\rangle_a |0\rangle_m$ , i.e., with the atom fully excited while the optical and the mechanical modes in the ground states, then during the evolution the system is confined to a *three-dimensional* Hilbert subspace, which allows for an analytical description of the coherent dynamics.

We note that the photon number plus the population in the atomic excited state is a constant of motion. Thus, starting from  $|0\rangle_c |e\rangle_a |0\rangle_m$ , the quantum state of the hybrid

system at any time  $t$  stays in the Hilbert subspace  $\{|1\rangle_c |g\rangle_a, |0\rangle_c |e\rangle_a\} \otimes \mathcal{H}_m$ , where  $|1\rangle_c$  is the one-photon Fock state, “ $\{\}$ ” stands for the subspace spanned by the basis vectors inside, and  $\mathcal{H}_m$  represents the Hilbert space of the mechanical mode. Assuming that the atom and the cavity are on resonance, i.e.,  $\omega_a = \omega_c$ , we construct the eigenstates of the atom-cavity part of the Hamiltonian  $[\hat{H}_{ca} = \omega_c \hat{c}^\dagger \hat{c} + \omega_a \hat{\sigma}_z/2 + g_{ca} (\hat{\sigma}_+ \hat{c} + \hat{\sigma}_- \hat{c}^\dagger)]$  in the two-dimensional subspace  $\{|1\rangle_c |g\rangle_a, |0\rangle_c |e\rangle_a\}$  as

$$|\pm\rangle_{ca} = \frac{|1\rangle_c |g\rangle_a \pm |0\rangle_c |e\rangle_a}{\sqrt{2}}, \quad (2)$$

which correspond to the eigenvalues of  $\omega_a/2 \pm g_{ca}$ . In this new basis  $\hat{H}_{ca}$  is diagonalized, i.e.,

$$\hat{H}_{ca} = \frac{\omega_a}{2} + g_{ca} \hat{\sigma}'_z, \quad (3)$$

with  $\hat{\sigma}'_z = |+\rangle_{ca} \langle +| - |-\rangle_{ca} \langle -|$ . Moreover, since  $\hat{c}^\dagger \hat{c} |\pm\rangle_{ca} = (|+\rangle_{ca} + |-\rangle_{ca})/2$ , one has  $\hat{c}^\dagger \hat{c} = (1 + \hat{\sigma}'_+ + \hat{\sigma}'_-)/2$ , where  $\hat{\sigma}'_+ = \hat{\sigma}'_-^\dagger = |+\rangle_{ca} \langle -|$ . Thus in the subspace  $\{|1\rangle_c |g\rangle_a, |0\rangle_c |e\rangle_a\} \otimes \mathcal{H}_m$ , the total Hamiltonian (1) becomes  $\hat{H} = \omega_a/2 + g_{ca} \hat{\sigma}'_z + \omega_m \hat{b}^\dagger \hat{b} - g_{cm} (1 + \hat{\sigma}'_+ + \hat{\sigma}'_-) (\hat{b} + \hat{b}^\dagger)/2$ . By introducing the displaced phonon operator  $\hat{b}' = \hat{b} - g_{cm}/(2\omega_m)$  the Hamiltonian is re-written as

$$\hat{H} = g_{ca} \hat{\sigma}'_z + \omega_m \hat{b}'^\dagger \hat{b}' - \frac{g_{cm}}{2} (\hat{\sigma}'_+ + \hat{\sigma}'_-) (\hat{b}' + \hat{b}'^\dagger) - \frac{g_{cm}^2}{2\omega_m} (\hat{\sigma}'_+ + \hat{\sigma}'_-),$$

where a constant energy term  $\omega_a/2 - g_{cm}^2/(4\omega_m)$  has been discarded. Further assuming that  $g_{cm} \ll \omega_m$  and  $\omega_m \approx 2g_{ca}$ , we neglect the term proportional to  $g_{cm}^2$  and make the rotating wave approximation in the above Hamiltonian to obtain  $\hat{H} \approx \hat{H}_{eff}$ , with

$$\hat{H}_{eff} = g_{ca} \hat{\sigma}'_z + \omega_m \hat{b}'^\dagger \hat{b}' - \frac{g_{cm}}{2} (\hat{\sigma}'_+ \hat{b}' + \hat{\sigma}'_- \hat{b}'^\dagger). \quad (4)$$

This Hamiltonian describes a two-level “polariton” coupled to a bosonic mode, analogous to the original atom-cavity Hamiltonian  $\hat{H}_{ca}$ .

### 3. Sinusoidally Modulated Rabi Oscillation

In the previous section, we derived an effective Hamiltonian  $\hat{H}_{eff}$  in (4) for the hybrid system where initially the atom is fully excited and both the optical and the mechanical modes are in their ground states. At first glance, since  $\hat{H}_{eff}$  is a J-C type Hamiltonian, it might suggest a simple Rabi oscillation of frequency  $g_{cm}$  between *two* basis vectors, analogous to that in a regular atom-cavity system (i.e., without the mechanical resonator). This would be true if the hybrid system were initially in  $|+\rangle_{ca} |0\rangle_{m'}$ , with  $|0\rangle_{m'}$  being the ground state of the displaced mechanical mode  $\hat{b}'$ . However, the initial state of the system is not  $|+\rangle_{ca} |0\rangle_{m'}$ , but rather  $|0\rangle_c |e\rangle_a |0\rangle_m$ . As will become clear below, starting from  $|0\rangle_c |e\rangle_a |0\rangle_m$ , the evolution of the hybrid system involves *three* (rather than two) basis vectors, which in turn leads to a non-trivial modulation of the original atom-cavity Rabi oscillation by the optomechanical coupling.

We first take a deeper look at the initial state  $|0\rangle_c |e\rangle_a |0\rangle_m$ . Although rigorously the ground state  $|0\rangle_m$  of the original mechanical mode  $\hat{b}$  is the coherent state of the displaced mode  $\hat{b}'$  with the eigenvalue  $-g_{cm}/(2\omega_m)$ , we can well approximate  $|0\rangle_m \approx |0\rangle_{m'}$  since it has been assumed that  $g_{cm} \ll \omega_m$ . According to (2),  $|0\rangle_c |e\rangle_a$  is a superposition of  $|+\rangle_{ca}$  and  $|-\rangle_{ca}$ , and thus the initial state  $|0\rangle_c |e\rangle_a |0\rangle_m \approx |0\rangle_c |e\rangle_a |0\rangle_{m'}$  is a superposition of  $|+\rangle_{ca} |0\rangle_{m'}$  and  $|-\rangle_{ca} |0\rangle_{m'}$ . Further noting that  $|-\rangle_{ca} |0\rangle_{m'}$  is an eigenstate of  $\hat{H}_{eff}$ , and  $\hat{H}_{eff}$  couples  $|+\rangle_{ca} |0\rangle_{m'}$  only to  $|-\rangle_{ca} |1\rangle_{m'}$  and vice versa, we conclude that the coherent dynamics of the system is constrained to the three-dimensional Hilbert subspace of

$$\mathcal{H}^{(3)} = \{|-\rangle_{ca} |0\rangle_{m'}, |+\rangle_{ca} |0\rangle_{m'}, |-\rangle_{ca} |1\rangle_{m'}\}, \quad (5)$$

which allows us to obtain an analytic solution for the evolution of the hybrid system.

To this end, we diagonalize  $\hat{H}_{eff}$  in the subspace  $\mathcal{H}^{(3)}$

$$\hat{H}_{eff} = -g_{ca} |\tilde{0}\rangle \langle \tilde{0}| + \Omega_+ |\tilde{+}\rangle \langle \tilde{+}| + \Omega_- |\tilde{-}\rangle \langle \tilde{-}|, \quad (6)$$

where

$$\begin{aligned} |\tilde{0}\rangle &= |-\rangle_{ca} |0\rangle_{m'}, \quad |\tilde{+}\rangle = \mathcal{N} (|-\rangle_{ca} |1\rangle_{m'} - \eta |+\rangle_{ca} |0\rangle_{m'}), \\ |\tilde{-}\rangle &= \mathcal{N} (\eta |-\rangle_{ca} |1\rangle_{m'} + |+\rangle_{ca} |0\rangle_{m'}), \end{aligned} \quad (7)$$

and  $\Omega_{\pm} = (\omega_m \pm \Omega_g)/2$ ,  $\Omega_g = \sqrt{(\omega_m - 2g_{ca})^2 + g_{cm}^2}$ ,  $\eta = [\Omega_g - (\omega_m - 2g_{ca})]/g_{cm}$ ,  $\mathcal{N} = (1 + \eta^2)^{-1/2}$ . With the initial state of the system decomposed [by inverting (2) and (7)] in terms of the three eigenvectors of  $\hat{H}_{eff}$  as

$$|\psi(t=0)\rangle = |0\rangle_c |e\rangle_a |0\rangle_{m'} = -\frac{\mathcal{N} (\eta |\tilde{+}\rangle - |\tilde{-}\rangle) + |\tilde{0}\rangle}{\sqrt{2}},$$

we get the quantum state at any time  $t$

$$|\psi(t)\rangle = -\frac{\mathcal{N}}{\sqrt{2}} \eta e^{-i\frac{\omega_m + \Omega_g}{2}t} |\tilde{+}\rangle + \frac{\mathcal{N}}{\sqrt{2}} e^{-i\frac{\omega_m - \Omega_g}{2}t} |\tilde{-}\rangle - \frac{1}{\sqrt{2}} e^{ig_{ca}t} |\tilde{0}\rangle.$$

To calculate the probability for the atom to stay in the excited state, we convert  $|\tilde{\pm}\rangle$  and  $|\tilde{0}\rangle$  in  $|\psi(t)\rangle$  back to the “original” basis vectors of  $|0\rangle_c |e\rangle_a |0\rangle_{m'}$ ,  $|1\rangle_c |g\rangle_a |0\rangle_{m'}$ ,  $|0\rangle_c |e\rangle_a |1\rangle_{m'}$  and  $|1\rangle_c |g\rangle_a |1\rangle_{m'}$ , which can be done by simply substituting (2) into (7). Here we do not present the detailed expression of  $|\psi(t)\rangle$  in the original basis, but only write down its probability amplitudes  $\mathcal{A}_{0e1}$  for  $|0\rangle_c |e\rangle_a |1\rangle_{m'}$  and  $\mathcal{A}_{0e0}$  for  $|0\rangle_c |e\rangle_a |0\rangle_{m'}$ :

$$\begin{aligned} \mathcal{A}_{0e1} &= -i\mathcal{N}^2 \eta e^{-i\frac{\omega_m}{2}t} \sin\left(\frac{\Omega_g}{2}t\right), \\ \mathcal{A}_{0e0} &= \frac{\mathcal{N}^2 e^{-i\frac{\omega_m}{2}t} \left(\eta^2 e^{-i\frac{\Omega_g}{2}t} + e^{i\frac{\Omega_g}{2}t}\right) + e^{ig_{ca}t}}{2}. \end{aligned}$$

The probability  $P_e$  to find the atom in the excited state is given by  $|\mathcal{A}_{0e1}|^2 + |\mathcal{A}_{0e0}|^2$ . In terms of the more widely used population inversion  $\Delta P = P_e - P_g$ , one has

$$\Delta P = \cos(\Omega_{ca}t) \cos \frac{\Omega_g t}{2} + \xi \sin(\Omega_{ca}t) \sin \frac{\Omega_g t}{2}, \quad (8)$$

where

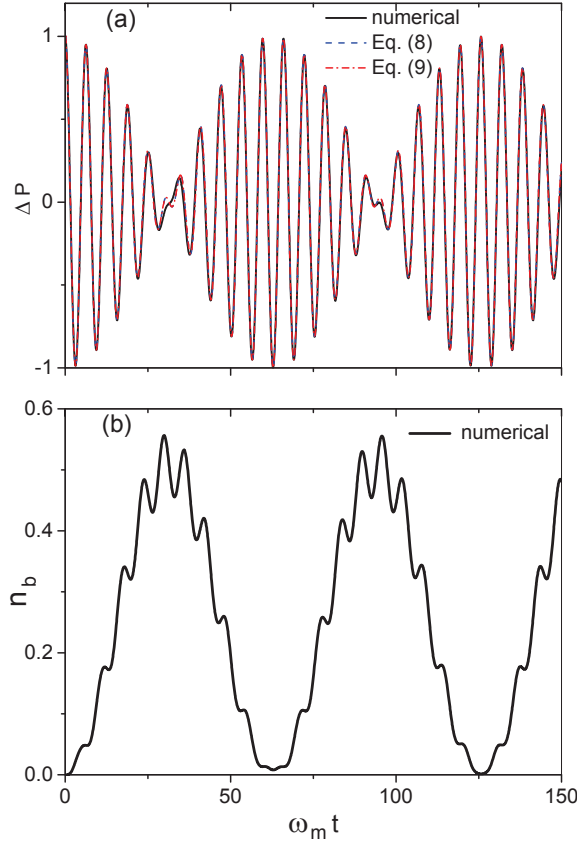
$$\Omega_{ca} = \frac{\omega_m}{2} + g_{ca}, \quad \xi = \frac{1 - \eta^2}{1 + \eta^2},$$

and  $P_g = 1 - P_e$  is the probability for the atom to be in the ground state. Eq. (8) is drastically different from the standard Rabi oscillation  $\Delta P_{ca} = \cos(2g_{ca}t)$  in the atom-cavity system governed by Hamiltonian  $\hat{H}_{ca}$  [(3)]. Particularly, if the atom-cavity Rabi splitting is on resonance with the mechanical mode, i.e.,  $\omega_m = 2g_{ca}$ , we can simplify (8) into

$$\Delta P = \cos(2g_{ca}t) \cos\left(\frac{g_{cm}}{2}t\right), \quad (9)$$

which shows that *the optomechanical coupling modulates the original atom-cavity Rabi oscillation with a cosine envelope function*. This sinusoidally modulated Rabi oscillation is our central result, and its derivation has involved a fairly large amount of mathematics, so in next paragraph we will provide a more physically intuitive argument.

To do this, we analyze the action of  $\hat{H}_{eff}$  in (4) on the three dimensional Hilbert subspace  $\mathcal{H}^{(3)}$  in (5). We note that  $\hat{H}_{eff}$  induces two distinct oscillatory transitions: on a short time scale of  $g_{ca}^{-1}$ , the atom-cavity interaction ( $g_{ca}\hat{\sigma}_z'$ ) yields the Rabi oscillation between  $|0\rangle_c |e\rangle_a |0\rangle_{m'} = (|+\rangle_{ca} - |-\rangle_{ca}) |0\rangle_{m'}/\sqrt{2}$  and  $|1\rangle_c |g\rangle_a |0\rangle_{m'} = (|+\rangle_{ca} + |-\rangle_{ca}) |0\rangle_{m'}/\sqrt{2}$ , or equivalently, the oscillation of the dynamical phase difference between the  $|+\rangle_{ca} |0\rangle_{m'}$  and  $|-\rangle_{ca} |0\rangle_{m'}$  components; on a long time scale of  $g_{cm}^{-1}$ , the optomechanical coupling  $[-g_{cm}(\hat{\sigma}_+' \hat{b}' + \hat{\sigma}_-' \hat{b}'^\dagger)/2]$  causes the “polariton-phonon” oscillation between  $|+\rangle_{ca} |0\rangle_{m'}$  and  $|-\rangle_{ca} |1\rangle_{m'}$ . At  $t = 0$ , the system is in  $|0\rangle_c |e\rangle_a |0\rangle_{m'} = (|+\rangle_{ca} - |-\rangle_{ca}) |0\rangle_{m'}/\sqrt{2}$ , and the atom-cavity interaction generates the Rabi oscillation on the short time scale. After a duration of  $\pi/g_{cm}$ ,  $|+\rangle_{ca} |0\rangle_{m'}$  evolves into  $|-\rangle_{ca} |1\rangle_{m'}$  due to the optomechanical coupling, and the state of the system becomes  $|-\rangle_{ca} (|1\rangle_{m'} + e^{i\phi} |0\rangle_{m'})/\sqrt{2}$ , where  $\phi$  is a phase difference between the two Fock states of the (displaced) mechanical mode. Around this time ( $t \approx \pi/g_{cm}$ ) there is no atom-cavity Rabi oscillation since the  $|+\rangle_{ca} |0\rangle_{m'}$  component does not exist in the system state. After another duration of  $\pi/g_{cm}$ , i.e., at  $t = 2\pi/g_{cm}$ ,  $|-\rangle_{ca} |1\rangle_{m'}$  evolves back to  $|+\rangle_{ca} |0\rangle_{m'}$ , and the atom-cavity Rabi oscillation re-appears. For other  $t \in (0, 2\pi/g_{cm})$ , partial Rabi oscillations (i.e., without completely reaching the ground or the excited states of the atom) occur. This pattern repeats in every period of  $2\pi/g_{cm}$ , leading to the periodically-modulated Rabi oscillation in (9). We notice that the envelope function in (9) has a period of  $4\pi/g_{cm}$ , but the Rabi oscillation on the short time scale does not distinguish positive and negative values of the envelope function, and thus the “modulation period” is indeed  $2\pi/g_{cm}$ , agreeing with our analysis above. We further remark that the involvement of three (rather than two) basis vectors in  $\mathcal{H}^{(3)}$  is crucial to the sinusoidally modulated Rabi oscillation, because it is the optomechanical transition between  $|+\rangle_{ca} |0\rangle_{m'}$  and  $|-\rangle_{ca} |1\rangle_{m'}$  that gives rise to the modulation of the regular atom-cavity Rabi oscillation in the subspace of  $\{|+\rangle_{ca} |0\rangle_{m'}, |-\rangle_{ca} |0\rangle_{m'}\}$ .



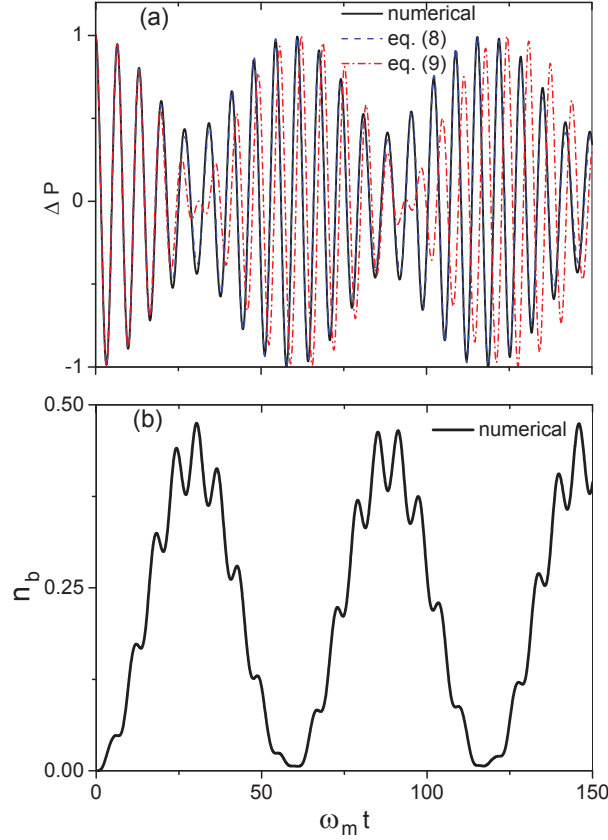
**Figure 2.** Temporal evolution of (a) the atomic population inversion, and (b) the mean phonon number, with  $g_{cm} = 0.1\omega_m$ ,  $\omega_a = \omega_c$ ,  $g_{ca} = 0.5\omega_m$ , and no thermal effects. The three curves in (a) are almost indistinguishable from each other, and the sinusoidally modulated Rabi oscillation is nearly perfect.

#### 4. Numerical Simulations and Non-Ideal Situations

Eq. (9), which illustrates the sinusoidal modulation of the Rabi oscillation in the atom-optomechanical system, has been derived with the following assumptions: no dissipations,  $\omega_c = \omega_a$ ,  $\omega_m = 2g_{ca}$ ,  $g_{cm} \ll \omega_m$ , and deviations from these ideal conditions are expected to affect the dynamics. For example, if  $\omega_m$  is slightly detuned from  $2g_{ca}$  (and thus  $\xi \neq 0$ ), then  $\Delta P$  is determined by Eq. (8) [rather than Eq. (9)], where the second term on the right-hand side perturbs the “ideal” dynamics manifested in the first term. To more comprehensively understand the robustness of the modulated Rabi oscillation, we resort to numerical simulations of the density-matrix ( $\hat{\rho}$ ) master equation [19] for the hybrid system

$$\frac{d\hat{\rho}}{dt} = \frac{1}{i} [\hat{H}, \hat{\rho}] - \kappa L[\hat{a}] \hat{\rho} - \gamma L[\hat{\sigma}_-] \hat{\rho} - (n_{th} + 1) \mu L[\hat{b}] \hat{\rho} - n_{th} \mu L[\hat{b}^\dagger] \hat{\rho}, \quad (10)$$

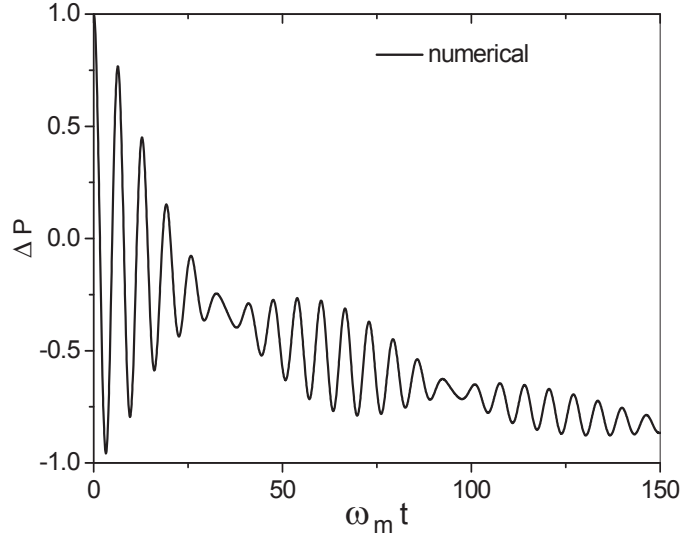
where  $\hat{H}$  is the unapproximated Hamiltonian in (1),  $L[\hat{o}] \hat{\rho} = \hat{o}^\dagger \hat{o} \hat{\rho} / 2 - \hat{o} \hat{\rho} \hat{o}^\dagger + \hat{\rho} \hat{o}^\dagger \hat{o} / 2$ ,  $\kappa$ ,  $\gamma$ ,  $\mu$  are the decay rates of the optical cavity, the atom, and the mechanical resonator, respectively, and  $n_{th}$  is the thermal occupation of the mechanical heat-bath mode at



**Figure 3.** Temporal evolution of (a) the atomic population inversion, and (b) the mean phonon number, with  $g_{cm} = 0.1\omega_m$ ,  $\omega_a = \omega_c$ ,  $g_{ca} = 0.48\omega_m$ , and no thermal effects. In (a), the curve for Eq. (8) essentially coincides with the numerical one, which shows a non-ideal modulated Rabi oscillation.

frequency  $\omega_m$ . It has also been assumed that both the optical and the atomic heat baths have zero thermal occupation at  $\omega_c$  and  $\omega_a$ , which is a good approximation at room temperature for  $\omega_c$  and  $\omega_a$  within the visible-light frequency range. The population inversion of the atom is calculated from the density matrix via  $\Delta P = \text{Tr}[\hat{\rho}\hat{\sigma}_z]$ , with “Tr” being the trace of operators.

We first check the validity of our analytic model for the sinusoidally modulated Rabi oscillation. In each of Figs. 2a and 3a we plot three curves: Eqs. (8) [blue dash] and (9) [red dash-dot], and the numerical simulation of the master equation (10) [black solid] with  $\rho(t=0) = |0\rangle_c |e\rangle_a |0\rangle_{mm} \langle 0|_a \langle e|_c \langle 0|$  and  $\kappa = \gamma = \mu = 0$ . In Fig. 2a, we take the parameters to be  $\omega_a = \omega_c$ ,  $g_{ca} = 0.5\omega_m$ ,  $g_{cm} = 0.1\omega_m$ , for which Eq. (9) is equivalent to the more general Eq. (8). We find that the two analytic curves well fit the numerical one, and the system undergoes a perfect sinusoidally-modulated Rabi oscillation. In Fig. 3a, we set  $\omega_a = \omega_c$ ,  $g_{ca} = 0.48\omega_m$ ,  $g_{cm} = 0.1\omega_m$ . For these parameters, Eq. (9) is no longer valid since  $\omega_m \neq 2g_{ca}$ , but Eq. (8) still holds. In this case, Eq. (8) agrees well with the numerical result, and the modulation of the Rabi oscillation is not ideally sinusoidal. Fig. 2a and Fig. 3a confirm that our analytic model correctly describes the



**Figure 4.** Modulated Rabi oscillation in presence of thermal effects. The parameters are  $g_{cm} = 0.1\omega_m$ ,  $\omega_a = \omega_c - 0.01\omega_m$ ,  $g_{ca} = 0.49\omega_m$ ,  $\kappa = 0.02\omega_m$ ,  $\gamma = 0.005\omega_m$ ,  $\mu = 2 \times 10^{-4}\omega_m$ ,  $n_{th} = 10$ , and the initial thermal phonon number of the mechanical resonator is  $n_{th}^{(0)} = 0.5$ .

coherent dynamics of the hybrid system within the proper parameter range.

In Figs. 2b and 3b, we plot the numerical results for the temporal evolution of the phonon number  $n_b = \text{Tr} [\hat{\rho} \hat{b}^\dagger \hat{b}]$ , with the same parameters as in Figs. 2a and 3a, respectively. According to the analysis in the last paragraph of Sec. III, the Rabi-oscillation amplitude is maximum (minimum) when the phonon number is minimum (maximum). Comparing Fig. 2a and Fig. 2b, as well as Figs. 3a and 3b, immediately confirms this conclusion, and thus provides further support for our analytic model in the previous section.

Next we investigate the situation where dissipations are present by adopting  $\kappa = 0.02\omega_m$ ,  $\gamma = 0.005\omega_m$ ,  $\mu = 2 \times 10^{-4}\omega_m$ ,  $n_{th} = 10$ . We also allow that the other parameters do not rigorously meet (but not deviate too much from) the requirements for Eq. (9), with  $\omega_a = \omega_c - 0.1\omega_m$ ,  $g_{ca} = 0.49\omega_m$ , and  $g_{cm} = 0.1\omega_m$ . Moreover, since the mechanical mode can at best be *pre-cooled* to a small but finite mean occupation number  $n_{th}^{(0)} < n_{th}$ , we set the mechanical resonator to be initially in a thermal state (rather than the ground state) of  $n_{th}^{(0)} = 0.5$ . The numerical solution for  $\Delta P$ , as plotted in Fig. 4, unambiguously demonstrates that *the modulated Rabi oscillation survives* even though it is significantly degraded from the ideal one illustrated in Eq. (9) [or in Fig. 2a].

## 5. Summary

In summary, the quantum behavior of an excited two-level atom in a vacuum optomechanical cavity can be qualitatively modified by the mechanical resonator.



Particularly, if the atom-cavity Rabi splitting is on resonance with the mechanical mode, then the standard Rabi oscillation of the atom is sinusoidally modulated. The modulation originates from the “polariton-phonon” transition caused by the coupling between the optical field and the mechanical resonator. We explained this phenomenon with an analytic model in a three-dimensional Hilbert subspace of the hybrid system, which was further confirmed by numerical simulations of the density-matrix master equation. The modulated Rabi oscillation was found to be reasonably tolerant on thermal effects and non-idealized parameters.

## 6. Acknowledgement

We gratefully acknowledge financial supports from the National Natural Science Foundation of China (No. 11375081, 11347026), the Shandong Provincial Natural Science Foundation (No. ZR2013AL007, ZR2013AM012), and the Start-up Fund of Liaocheng University.

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